A Central Force Oscillating in Time Caused by a Linear Change in Time of the Magnetic Induction

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A charged particle in its planar motion in a constant magnetic field is submitted a centripetal force acting on that particle. The amplitude of this force is constant in time. In this paper it is demonstrated that by a linear change in time of the magnetic induction another centripetal force acting on the particle is created. This force has an amplitude oscillating in time with the frequency equal to that of the particle gyration.

Key words: Magnetic Induction; Time Dependent Central Force.

1. Introduction

The motion of a charged particle in a magnetic field is fundamental in electrodynamics. When the magnetic field is constant in space and time, the problem – both in classical and quantum mechanics – has its well-known solution; see e. g. [1]. However, mainly in astrophysics [2 – 4], space physics [5, 6], and plasma physics [7,8], there arises very often the problem of the motion of a charged particle when the intensity of the magnetic field depends on space and time. In such cases finding the motion problem is quite a complicated task. The complications concern not only the methods used for solving it, but also the fields themselves, since different kinds of fields are often combined for possible applications [9].

In the present paper we elaborate an example of such an integrable system by considering the mechanical force which arises when the intensity of a spatially uniform magnetic field is changed linearly in time. In spite of its apparent simplicity, this problem seems never to have been solved before, although the formal solutions necessary for describing the particle motion in a field of this kind are well known [10 – 12].

For a constant magnetic field having the induction $\vec{B}_0$ a charged particle is moving in a plane perpendicular to $\vec{B}_0$ with the constant angular velocity having the value

$$\dot{\varphi} \equiv \omega_{b0} = -\frac{eB_0}{mc}, \tag{1}$$

where e is the charge and m the mass of the particle. The angular momentum of the particle has the constant value

$$l = mr^2\dot{\varphi} \tag{2}$$

at a constant distance r from the gyration center.

A constant value of l represents a sufficient condition for the force acting on the particle to be central about the point which is fixed at the origin. This force, labelled by F_r, is acting on the particle moving in the r, φ-plane. F_r can be obtained directly from the equation [13, 14],

$$\frac{\mathrm{d}^2}{\mathrm{d}\varphi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{l^2}F_r. \tag{3}$$

For a constant field B_0 the radial coordinate r of the trajectory is independent of the polar angle φ; in this case we obtain from (3)

$$F_r = -\frac{l^2}{mr^3}. \tag{4}$$

The integral of F_r, done over r, gives the potential $V(r)$ which can be readily obtained from the non-relativistic Langrangian L presented in polar coordinates [15].

The solution of the problem is found, first, by solving the corresponding equation for $r(t)$ and, next, by substituting this result into the formula (3) for F_r. Our calculation is done on a strictly non relativistic basis; also no self-reaction (Bremsstrahlung) of the the field is taken into account. Only a slowly varying $\vec{B}$-field is allowed, giving an adiabatic change in the considered equations of motion.

2. Particle Trajectory Obtained in the Case of Temporal Change of the Magnetic Induction

A slow variation of $\vec{B}$ with t produces an electric field $\vec{E}$ according to the equation

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{5}$$

We assume that the direction of $\vec{B}$ is not changed in course of the change of B from $B = B_0$ at $t = 0$ to $B = B_0 + \Delta B$ at $t = t_{\max}$, and the field $\vec{B}$ remains parallel to the z- axis of the Cartesian coordinate system, so the system preserves its cylindrical symmetry. Consequently, $\vec{E}$ becomes zero along the z-axis which is the cylinder axis. Because $\vec{B}$ does not depend on x, y and z, we obtain (see e. g. [8])

$$\vec{E} = \frac{1}{2}\vec{r} \times \frac{\mathrm{d}\vec{B}}{\mathrm{d}t}. \tag{5a}$$

The change of $\vec{B}$ means a similar change of the angular frequency ω_{b0} into

$$\vec{\omega}_b = -\frac{e}{m}\vec{B}. \tag{6}$$

We put henceforth the light velocity $c = 1$. Hence the Lorentz acceleration

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}) \tag{7}$$

becomes

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \frac{1}{2}\frac{\mathrm{d}\vec{\omega}_b}{\mathrm{d}t} \times r + \vec{\omega}_b \times \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}. \tag{7a}$$

Since the particle motion takes place on a plane perpendicular to $\vec{B}$, the position vector of the particle $\vec{r}$ can be described in terms of the polar coordinates (r, φ), whereas the vector $\vec{\omega}_b$ and its time derivative $\mathrm{d}\vec{\omega}_b/\mathrm{d}t$ are directed along z-axis. By identyfying $\vec{r} = \vec{i}x + \vec{j}y$, and by putting next $z = x + iy = re^{i\varphi}$ [8], a fundamental equation for the length r of the position vector is obtained when the real and imaginary part of (7a) are taken into account [15–17]:

$$\ddot{r} - \frac{C^2}{r^3} + \frac{1}{4}\omega_b^2 r = 0. \tag{8}$$

Equation (8) is a well-known formula; its properties were widely discussed at various occasious (see e. g. [9, 18]). In our case we have at $t = 0$ the angular velocity $\dot{\varphi} = \omega_b = \omega_{b0}$ and $r = r_0$. Here ω_{b0} is a constant angular frequency given by (1), and r_0 is a constant radial coordinate at $B = B_0$. The constant

$$C = r^2\dot{\chi} = r^2\left(\dot{\varphi} - \frac{1}{2}\omega_b\right), \tag{9}$$

entering (8), is at $t = 0$ equal to

$$C = \frac{1}{2}r_0^2\omega_{b0}. \tag{9a}$$

ω_b in (8) and (9) is the absolute value of the time-dependent angular frequency.

Let us assume that during a process, the duration of which is $t_{\max}$, the field B is changing linearly with t. Consequently, the original frequency ω_{b0} rises linearly to some new frequency

$$\omega_b(t) = \omega_{b0} + \omega_{b1}\frac{t}{t_{\max}}. \tag{10}$$

From (1) and (6) we see that the change of the magnetic induction $B - B_0 = \Delta B(t)$ is proportional to the last term entering (10). At $t = t_{\max}$

$$\omega_b\,(t_{\max}) = \omega_{b0} + \omega_{b1}. \tag{10a}$$

In general we assume

$$\omega_{b1} \ll \omega_{b0}. \tag{11}$$

The adiabacity requirement for the particle motion (see e.g. [8]) imposes the conditions

$$\frac{\dot{\omega}_b}{\omega_b^2} \ll 1, \tag{12}$$

$$\frac{1}{\omega_b^3}\ddot{\omega} \ll 1. \tag{13}$$

Evidently, because of (10), (13) is satisfied for any value of ω_{b1}, $t_{\max}$ and t; relation (12) is satisfied because of (11).

The solution of (8) can be obtained by assuming that the modified r is not much different from r_0, the radius at $t = 0$, and put (see e.g. [14])

$$r = r_0 + u = r_0\left(1 + \frac{u}{r_0}\right), \tag{14}$$

where $u \ll r_0$. Since r_0 is a constant, we obtaim $\ddot{r} = \ddot{u}$.

In consequence, (8) can be linearized into the equation

$$\ddot{u}+au+bt=-cut-f(r_0+u)t^2\approx 0, \qquad (15)$$

where

$$a=\omega_{b0}^2, \quad b=\frac{1}{2}\frac{\omega_{b0}\omega_{b1}r_0}{t_{\max}}, \qquad (16)$$

because, in view of (11), we have

$$cu\ll b, \quad f(r_0+u)t^2\ll bt. \qquad (17)$$

The solution of the resulting differential equation (15) can be obtained analytically [16]:

$$u=\frac{1}{2}\frac{\omega_{b1}r_0}{\omega_{b0}t_{\max}}\left[-t+\frac{\sin(\omega_{b0}t)}{\omega_{b0}}\right]. \qquad (18)$$

The boundary conditions, satisfying (18), are $u(0)=0$, $\dot{u}(0)=0$. In Sect. 3 we apply (18) to the calculation of the force acting on the particle.

3. Angular Velocity and Calculation of the Force Acting on a Charged Particle

Basing on [16] and (18), the angular velocity obtained for the particle motion in the plane becomes [see (9)]

$$\dot{\varphi}=\frac{C}{r^2}+\frac{1}{2}\omega_b=\frac{r_0^2}{r^2}\frac{\omega_{b0}}{2}+\frac{1}{2}\left(\omega_{b0}+\frac{\omega_{b1}t}{t_{\max}}\right) \qquad (19)$$

$$\approx \omega_{b0}+\frac{1}{2}\frac{\omega_{b1}}{t_{\max}}t-\frac{u\omega_{b0}}{r_0}$$

because of (14). In effect by (18) and (19) we get

$$\dot{\varphi}=\omega_{b0}+\frac{\omega_{b1}t}{t_{\max}}-\frac{1}{2}\frac{\omega_{b1}}{\omega_{b0}t_{\max}}\sin(\omega_{b0}t). \qquad (20)$$

The last term in (20) is an oscillating expression with an amplitude which is very small compared to the second term on the right-hand side of (20).

This condition is satisfied for any $t_{\max}$ extended over many gyroperiods $T_{b0}=2\pi\omega_{b0}^{-1}$. In effect,

$$\dot{\varphi}=\omega_b \qquad (21)$$

given in (10). By substituting (21) into (9) we obtain the relation

$$C=\frac{1}{2}r^2\dot{\varphi}. \qquad (22)$$

This step demonstrates independence of the angular momentum $l=2mC$ on time and proves the central character of the force acting along the coordinate r.

From (22) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}\varphi}=\frac{\mathrm{d}t}{\mathrm{d}\varphi}\frac{\mathrm{d}}{\mathrm{d}t}=\frac{r^2}{2C}\frac{\mathrm{d}}{\mathrm{d}t}, \qquad (23)$$

so

$$\frac{\mathrm{d}}{\mathrm{d}\varphi}\left(\frac{1}{r}\right)=\frac{\mathrm{d}}{\mathrm{d}\varphi}\left(\frac{1}{r_0+u}\right)\left(\frac{1}{r}\right)=\frac{r^2}{2C}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{r_0+u}\right) \qquad (24)$$

$$\cong -\frac{r^2}{2Cr_0^2}\frac{\mathrm{d}u}{\mathrm{d}t}=\frac{r^2\omega_{b1}r_0}{2Cr_0^2\omega_{b0}t_{\max}}\sin^2\left(\frac{\omega_{b0}t}{2}\right)$$

because of (20) and the fact only a small part of r, labeled by u, depends on t in an efficient way. The next differentiation gives

$$\frac{\mathrm{d}^2}{\mathrm{d}\varphi^2}\left(\frac{1}{r}\right)=\left(\frac{r^2}{2C}\right)^2\frac{1}{r_0^2}\frac{\omega_{b1}r_0}{\omega_{b0}t_{\max}}\frac{\mathrm{d}}{\mathrm{d}t}\sin^2\left(\frac{\omega_{b0}t}{2}\right)$$

$$=\frac{1}{2}\frac{r^4}{r_0^4}\frac{\omega_{b1}}{\omega_{b0}^2r_0t_{\max}}\sin(\omega_{b0}t) \qquad (25)$$

$$\approx\frac{1}{2}\frac{\omega_{b1}}{\omega_{b0}^2r_0t_{\max}}\sin(\omega_{b0}t).$$

In the calculation of (25) we used the formula (9a) for C and also the property of $r/r_0\approx 1$. A substitution of (25) into (3) gives

$$\frac{1}{2}\frac{\omega_{b1}}{\omega_{b0}^2}\frac{\sin(\omega_{b0}t)}{t_{\max}r_0}+\frac{1}{r}=-\frac{r^2}{(2C)^2}F_r, \qquad (26)$$

since C equals half of the value of the angular momentum; see (22). In (26), and henceforth, we put m = 1.

We see that the centripetal force, given by the term r^{-1} entering the left-hand side of (26), equals

$$-\frac{(2C)^2}{r^3}=F_r(B_0) \qquad (27)$$

calculated in (4). This force is supplemented by the new force

$$-2C^2\frac{\omega_{b1}}{\omega_{b0}^2}\frac{\sin(\omega_{b0}t)}{t_{\max}r_0r^2}=F_r[\Delta B(t)], \qquad (28)$$

so

$$F_r(B_0)+F_r[\Delta B(t)]=F_r \qquad (29)$$

is the total force acting on the particle. Since F_r and $F_r(B_0)$ are central, the same property holds for the component $F_r[\Delta B(t)]$.

As far as we know, the supplementary force $F_r[\Delta B(t)]$ obtained in (28) is a novel result. The force oscillates sinusoidally in time with an amplitude approximately proportional to r^{-3}. Obviously, for $\omega_{b1} \to 0$ we have

$$F_r[\Delta B(t)] \to 0, \tag{30}$$

so the force (28) vanishes with vanishing $\Delta B(t)$, which is proportional to ω_{b1}, see (1), (6) and (10). We show in Sect. 4 that the result (25) can be derived also on the basis of the solution of a more accurate equation than that given by the last step of (15).

4. Calculation of the Force Done on the Basis of an Exact Equation (15)

Our aim is to improve the solution (18) by constructing a new solution which takes into account the terms neglected in (15). We assume

$$u = u_0 + w, \tag{31}$$

where u_0 is equal identically to (18). We seek now a function w which – we expect – is a small correction of u_0.

A substitution of (31) into (15) gives

$$\ddot{w} + \left(\omega_{b0}^2 + \frac{1}{2}\frac{\omega_{b0}\omega_{b1}}{t_{\max}}t\right)w = \frac{1}{4}\frac{\omega_{b1}^2}{\omega_{b0}}\frac{r_0 t}{t_{\max}^2}\sin(\omega_{b0}t). \tag{32}$$

It is easy to check that a special solution of (32) is

$$w^s = -\frac{1}{2}\frac{\omega_{b1}}{\omega_{b0}^2}\frac{r_0}{t_{\max}}\sin(\omega_{b0}t), \tag{33}$$

whereas the homogeneous equation (32) becomes

$$\ddot{w} + (a + ct)w = 0. \tag{34}$$

This equation can be reduced to the formula [19, 20]

$$\frac{\mathrm{d}^2\eta(\xi)}{\mathrm{d}\xi^2} + \frac{\xi\eta}{c^2} = 0, \tag{35}$$

where we have put

$$w(t) = \eta(\xi) \tag{36}$$

and $\xi = a + ct$; $c \neq 0$. A general solution of (35) is [19, 20]

$$\eta^g = \xi^{1/2}\left[A_1 J_{1/3}\left(\frac{2}{3c}\xi^{3/2}\right) + A_2 J_{-1/3}\left(\frac{2}{3c}\xi^{3/2}\right)\right], \tag{37}$$

where A_1 and A_2 are arbitrary constants and $J_{1/3}$ and $J_{-1/3}$ are Bessel functions of the first kind and the order $\frac{1}{3}$ and $-\frac{1}{3}$, respectively.

The solutions (33) and (37) can be combined. Since u_0 satisfies the initial conditions listed, our aim is to satisfy similar initial conditions for a full solution (31) at $t = 0$.

An important point is that the special solution w^s given in (33) cancels exactly the second term of $u = u_0$ given in (18). In effect, the extended solution becomes

$$\begin{aligned} u_0 + w = &-\frac{1}{2}\frac{\omega_{b1}r_0}{\omega_{b0}t_{\max}}t + A_1\xi^{1/2}J_{1/3}\left(\frac{2}{3c}\xi^{3/2}\right) \\ &+ A_2\xi^{1/2}J_{-1/3}\left(\frac{2}{3c}\xi^{3/2}\right). \end{aligned} \tag{38}$$

This result – together with the initial conditions for (31) – gives us the required equations for A_1 and A_2.

The result of the operator $\mathrm{d}^2/\mathrm{d}\varphi^2$ acting on

$$\frac{1}{r} = \frac{1}{r_0 + u_0 + w} \cong \frac{1}{r_0}\left(1 - \frac{u_0}{r_0} - \frac{w}{r_0}\right) \tag{39}$$

can be represented by

$$\begin{aligned} \frac{\mathrm{d}^2}{\mathrm{d}\varphi^2}\left(\frac{1}{r}\right) &= \left(\frac{\mathrm{d}t}{\mathrm{d}\varphi}\right)^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{1}{r}\right) \\ &\cong -\left(\frac{\mathrm{d}t}{\mathrm{d}\varphi}\right)^2\frac{1}{r_0^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}(u_0 + w). \end{aligned} \tag{40}$$

The reciprocal expression of the first-derivative term entering (40), namely

$$\begin{aligned} \frac{\mathrm{d}\varphi}{\mathrm{d}t} \equiv \dot{\varphi} = \frac{C}{r^2} + \frac{1}{2}\omega_b = \frac{r_0^2}{r^2}\frac{\omega_{b0}}{2} + \frac{1}{2}\left(\omega_{b0} + \frac{\omega_{b1}t}{t_{\max}}\right) \\ \cong \omega_{b0} + \frac{1}{2}\frac{\omega_{b1}}{t_{\max}}t \cong \omega_{b0}, \end{aligned} \tag{41}$$

is approximately a constant, since $r_0 \cong r$ and the term $\omega_{b1}t/t_{\max} \leq \omega_{b1}$ is small in comparison to ω_{b0}. In this way the dependence on t is represented mainly by the second time derivative of the expression (38) entering (40):

$$\frac{d^2}{dt^2}(u_0+w) = \frac{d^2}{dt^2}\left[A_1\xi^{1/2}J_{1/3}\left(\frac{2}{3c}\xi^{3/2}\right)+A_2\xi^{1/2}J_{-1/3}\left(\frac{2}{3c}\xi^{3/2}\right)\right] = -A_1\xi^{3/2}J_{1/3}\left(\frac{2}{3c}\xi^{3/2}\right)-A_2\xi^{3/2}J_{-1/3}\left(\frac{2}{3c}\xi^{3/2}\right) \tag{42}$$

The result presented in (42) can be made more transparent if we note that the argument of the Bessel functions is equal to

$$x=\frac{2}{3c}\xi^{3/2}\cong\frac{4}{3}t_{\max}\frac{\omega_{b0}^2}{\omega_{b1}}+\omega_{b0}t. \tag{43}$$

For not too small $t_{\max}$ this is a very large number because of (11), and a large frequency number ω_{b0}. This property allows us to apply asymptotic expansions for $J_{1/3}(x)$ and $J_{-1/3}(x)$ [21]. When susbstituted into (42), they give approximately

$$\ddot{u}_0+\ddot{w} = -A_1\frac{3^{1/2}}{(2\pi)^{1/2}}\left(\frac{\omega_{b1}}{t_{\max}}\right)^{1/2}\omega_{b0}^2\cos\left(\omega_{b0}t+\delta-\frac{1}{6}\pi\right) -A_2\frac{3^{1/2}}{(2\pi)^{1/2}}\left(\frac{\omega_{b1}}{t_{\max}}\right)^{1/2}\omega_{b0}^2\cos\left(\omega_{b0}t+\delta+\frac{1}{6}\pi\right), \tag{44}$$

where

$$\delta=\frac{2}{3c}a^{3/2}-\frac{1}{4}\pi=\frac{4}{3}\frac{\omega_{b0}^2t_{\max}}{\omega_{b1}}-\frac{1}{4}\pi \tag{45}$$

is a dimensionless parameter.

The expression (44) is a function oscillating in time with a frequency ω_{b0}. Our task is now to calculate the constants A_1 and A_2. The first initial condition, represented by the first equation given below (18) applied to u_0+w instead of u, can be reduced to

$$A_1\cos\left(\delta-\frac{1}{6}\pi\right)=-A_2\cos\left(\delta+\frac{1}{6}\pi\right), \tag{46}$$

whereas the second initial condition below (18) applied to u_0+w becomes approximately

$$-\frac{1}{2}\frac{\omega_{b1}r_0}{\omega_{b0}t_{\max}}-a\left[A_1\left(\frac{3c}{\pi a^{3/2}}\right)^{1/2}\cos\left(\delta-\frac{1}{6}\pi\right) + A_2\left(\frac{3c}{\pi a^{3/2}}\right)^{1/2}\cos\left(\delta+\frac{1}{6}\pi\right)\right]=0. \tag{46a}$$

For

$$\delta=\delta_1=\pm2\pi n, \tag{47}$$

where n is an integer, we obtain $A_1=-A_2$ from (46), whereas for

$$\delta=\delta_2=\left(\frac{1}{2}\pm2n\right)\pi \tag{47a}$$

(46) gives $A_1=A_2$. This implies that, for many cases of the phase shift δ, we have $|A_1|\simeq|A_2|$. The arithmetical mean of δ_1 and δ_2 given in (47) and (47a) is

$$\bar{\delta}=\left(\frac{1}{4}\pm2n\right)\pi. \tag{48}$$

For this $\bar{\delta}$ we have $\cos\bar{\delta}=\sin\bar{\delta}=2^{-1/2}$, and from the condition (46) we obtain

$$A_1\left(\frac{3^{1/2}}{2}+\frac{1}{2}\right)=-A_2\left(\frac{3^{1/2}}{2}-\frac{1}{2}\right); \tag{49}$$

for the same $\bar{\delta}$ the condition (46a) – together with the result of (49) – gives

$$-\frac{\omega_{b1}r_0}{\omega_{bo}t_{\max}}-\left(1+3^{1/2}\right)\left(\frac{6}{\pi}\right)^{1/2}a^{1/4}c^{1/2}A_1=0, \tag{50}$$

so we obtain

$$A_{1,2}=-\left(\frac{\omega_{b1}}{t_{\max}}\right)^{1/2}\frac{r_0}{\omega_{b0}^2}\left(\frac{\pi}{3}\right)^{1/2}\frac{1}{1\pm3^{1/2}}. \tag{51}$$

A substitution of A_1 and A_2 into (44) gives

$$\ddot{u}_0+\ddot{w}=-\frac{\omega_{b1}}{t_{\max}}r_0\frac{1}{2}3^{1/2}\sin(\omega_{b0}t), \tag{52}$$

where we have put $\delta=\bar{\delta}$ from (48). By taking into account (41), and substituting the result given in (52) into (40) we obtain

$$\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right)=\frac{1}{2}\frac{\omega_{b1}}{\omega_{b0}^2r_0}\frac{3^{1/2}}{t_{\max}}\sin(\omega_{b0}t). \tag{53}$$

Apart from the factor $3^{1/2}$, (53) is identical to (25) obtained before. Therefore, with the accuracy to a constant multiplier, the attained corrective force is the same as it has been calculated in (28).

Let us note that a comparison of (18) with the solution $r-r_0$ of the non- linear equation (8) obtained numerically for some chosen values of ω_{b1} and ω_{b0} has been done in [16], indicating a good approximation provided by the formula (18). The original ω_{b0} and r_0 taken into account were typical for the cyclotron resonance in metals [22].

5. Discussion

In the above calculations the gyration center of the moving particle is assumed to be at rest. But in fact this center performs small rotational motion for itself. The coordinates of the gyration center calculated for the present case of a time-dependent magnetic field are [16]

$$D_x = \frac{\omega_{b1}}{2\omega_{b0}^2 t_{\max}} r_0 \sin(\omega_{b0} t), \tag{54}$$

$$D_y = \frac{\omega_{b1}}{2\omega_{b0}^2 t_{\max}} r_0 [1 - \cos(\omega_{b0} t)], \tag{55}$$

Coordinates D_x and D_y represent a circle, the center of which is shifted along the axis of y by an amount equal to the circle radius

$$r_D = \frac{\omega_{b1} r_0}{2\omega_{b0}^2 t_{\max}}. \tag{56}$$

For any $t_{\max}\omega_{b0} \gg 1$ expression (55) has a very small value because of the property assumed in (11).

6. Summary

The paper examines the problem how the well-known centripetal force acting on a charged particle in a constant magnetic field is modified when this field is changing linearly in time. An advantage of the applied approach is, that for a slow change of the magnetic field the correcting centripetal force obtained for the original force could be calculated in an analytic manner.

A characteristic result of the paper is that a linear change in time of the magnetic field causes the amplitude of the correcting force to oscillate sinusoidally in time. The frequency of oscillation of that amplitude becomes equal to the original gyration frequency of the particle. Simultaneously, the amplitude of the correcting force becomes proportional to the rate of change of the particle gyration frequency divided by the square value of that frequency. This second property makes the correcting centripetal force exceedingly small for very large gyration frequencies.

The gyration center of the particle motion effectuates small rotational motion for itself. The parameters characteristic for this motion are similar to those of the correcting force. The gyration center oscillates with a frequency equal to that of the particle gyration, whereas the amplitude of the center oscillation is proportional to: (i) the rate of change of the gyration frequency of the particle due to the change of the magnetic field, (ii) the reciprocal square value of that frequency.

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